SYSTEMS CONSIDERATIONS FOR THERMOELECTRIC DESIGN

ABSTRACT

The figure-of-merit, Z, is often used as a predictor of thermoelectric generator performance without regard for the specific values of electrical conductivity, thermal conductivity or thermopower and without acknowledging the influence of system parasitics. The advantage of Z is that it is a single convenient metric against which different candidate thermoelectric materials may be measured. However, an exclusive reliance on Z to forecast the efficiency with which the energy in a heat flow can be converted to electrical energy can be misleading. This white paper describes some of the systems considerations that influence a successful deployment of thermoelectric generators. A similar analysis is valid for thermoelectric heat pumping.

INTRODUCTION

Thermoelectric generation is the solid state conversion of heat energy flux to electrical energy. A material's suitability for thermoelectric duties is embodied in three material transport properties, namely, electrical conductivity σ, thermal conductivity κ, and thermopower S. To analyze the maximum thermoelectric performance of a pellet of material, it is common to make the simplifying assumptions that the pellet has: (i) a uniform cross-sectional area, A; (ii) a uniform length, L; (iii) constant material properties, σ, κ and S; and (iv) an attached electrical load that matches the internal resistance of the thermoelectric pellet. This set-up is depicted in Figure 1, where the upper surface of the pellet has a uniform temperature, T_H, the bottom surface of the pellet has a uniform temperature T_C, and the heat energy flux is Q. In Figure 1, although the external load, R, is depicted as attaching to points on the top and bottom surfaces, the electrical attachments are actually to the entire top and bottom surfaces and are assumed to be zero resistance attachments. When the pellet is a P-type thermoelectric, the generated electrical current, I, will flow in the direction shown.

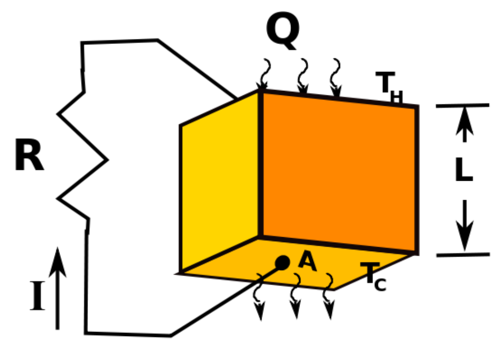

Figure 1 -- A Thermoelectric Pellet

Given the stated assumptions, the conversion efficiency of heat energy flux, Q, to generated electrical power, P, in the load, R, is [1]:

$$\aleph = \frac{P}{Q} = \frac{\Delta T}{T_H} \times \frac{\sqrt{1+ZT_{avg}}-1}{\sqrt{1+ZT_{avg}}+\frac{T_C}{T_H}} \qquad (1)$$

where $\Delta T = T_H - T_C$, $T_{avg} = (T_H + T_C)/2$ and thermoelectric figure-of-merit is defined as

$$Z = \frac{\sigma S^2}{k} \qquad (2)$$

From equation (1), it can be seen that the actual cross-sectional area, A, and length, L, are immaterial to conversion efficiency. Of particular significance is that the single variable, Z, is sufficient to express the conversion efficiency capability of a thermoelectric material -- that is, the transport properties, σ, κ and S lose their individual significance. With the stated assumptions, the maximum power generated by this idealized pellet is [1]:

$$P_{max} = \frac{\sigma S^2 \Delta T^2 A}{4L} \qquad (3)$$

where we note that dimensions A and L are now important, as is the product σS^2 (the aptly named "power factor"). However, thermal conductivity, κ, does not appear in this expression.

Although the single element model in Figure 1 is a convenient analysis tool, real generators are made from multiple thermoelements that are connected together. Relative to the single element ideal model, these connections always reduce the amount of power that can be generated per element and these connections always reduce the efficiency with which that power can be produced.

THE IMPACT OF CONNECTIONS

Figure 2 depicts a side view of the so-called π topology whereby multiple N-type and P-type thermoelements are connected in electrical series and in thermal parallel between two thermal reservoirs having constant temperatures T_H and T_C. The components that electrically connect adjacent elements are denoted in Figure 2 as "conductor" and are chosen to be both a good electrical conductor and a good thermal conductor. Examples are nickel and copper. The thermoelements are supported through physical scaffoldings which make up part of the labeled "substrates" in Figure 2 and serve to separate the thermoelectric generator from the thermal reservoirs. Besides the scaffoldings, there are always additional thermal resistances separating the top scaffolding from an infinite heat source (having temperature T_H) and separating the bottom scaffolding from an infinite heat sink (having temperature T_C). These thermal resistances may arise, for example, from fluid boundary layers separating the thermoelectric generator from an idealized source and sink, and in our discussion and analysis, we lump those effects into the substrate. These effects are always present and they always increase the effective thermal resistance separating the thermoelectric generator

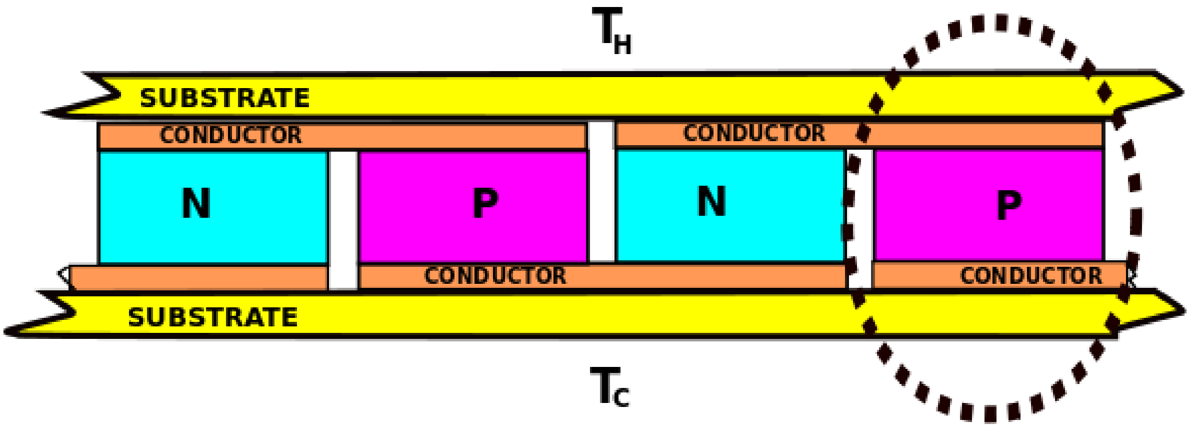

Figure 2 -- A PI Configuration Thermoelectric Generator

from ideal sources/sinks. For convenience, we also assume that the N-type and P-type thermoelements have the same electrical conductivity, thermal conductivity and the same magnitude of thermopower (with opposite signs). With these assumptions, we can analyze a single element, which is circled in Figure 2, and use this to guide the optimal design from the standpoint of power generation and conversion efficiency.

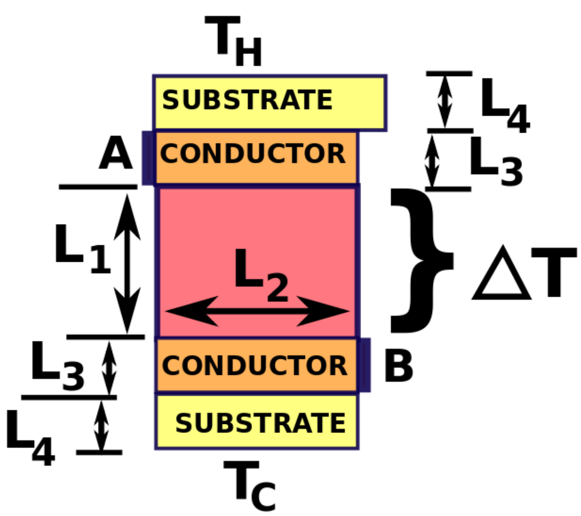

Figure 3 -- Detail of a Single Element

A detail of the side view of the circled thermoelement in Figure 2 is shown in Figure 3 and serves as the basis for a simulation study to determine the optimal design. A heat source on the top maintains a temperature T_H. Heat energy flux flows from the heat source through a top substrate, then through a top conductor into the top of the active thermoelement. The heat flows through the thermoelement, where some of the heat energy is converted to electrical energy and the balance of the heat energy flows out of the thermoelement through the bottom conductor and bottom substrate and into the heat sink which has fixed temperature T_C. The model for the top and bottom substrates incorporates all thermal resistances separating the thermoelectric generator from infinite reservoirs. The top and bottom substrates are modeled as having thickness L_4 and thermal conductivity κ_S.

The top and bottom conductors are modeled as having thickness L_3, and the active thermoelement has thickness L_1 and length L_2.

SIMULATION TO IDENTIFY THE IMPACT OF CONNECTIONS

A simulation study can be carried out using a finite element approach [3]. This approach can accommodate temperature dependent thermoelectric parameters, and, in particular, allows the expression of Thompson and Peltier effects. Although Figures 2 and 3 show a two dimensional side view, a thermoelectric device is a three dimensional apparatus. However, since the third axis (into the page) is perpendicular to the directions of flow of both the heat energy flux and the electrical current, this "depth" is arbitrarily chosen in all simulations to be 1 mm. Any determination of power will scale to the depth, so, for example, if a given device has 1 mm of depth and produces 1 mW of power, then a 2 mm deep device will produce 2 mW, etc. In all simulations, the substrate (the interface between thermal reservoirs and electrical conductors) is modeled as having a thickness of L_4=1 mm. For nickel conductors and two different P-type thermoelectric materials, a determination of the optimal thermoelement dimensions L_1, L_2 and L_3 was made.

For each dimension set, simulation yields the open circuit (no load) voltage between nodes A and B in Figure 3. Then nodes A and B are electrically shorted and the short circuit current is determined. Then the electrical load that will draw the maximum power is calculated as

$$R_{max} = \frac{V_{OC}}{I_{SC}} \qquad (4)$$

where V_{OC} and I_{SC} are, respectively, the open circuit voltage and the short circuit current. Finally, for each set (L_1, L_2, L_3), the calculated resistance, R_{max} is attached between terminals A and B and the resulting output power and conversion efficiency is derived through simulation.

RESULTS

A simulation study illustrates the way in which the various dimensions indicated in Figure 3 impact performance. For all experiments, we assume nickel conductors having constant values S=22 μV/K, σ=3.6e6 $\Omega^{-1}m^{-1}$ and κ=57 W/mK. For all experiments, the heat source and sink temperatures are, respectively, T_H=850 K and T_C=550 K. For the first set of experiments, the thermoelectric is chosen as P-type bulk SiGe material having temperature dependent transport properties shown in Figure 4.

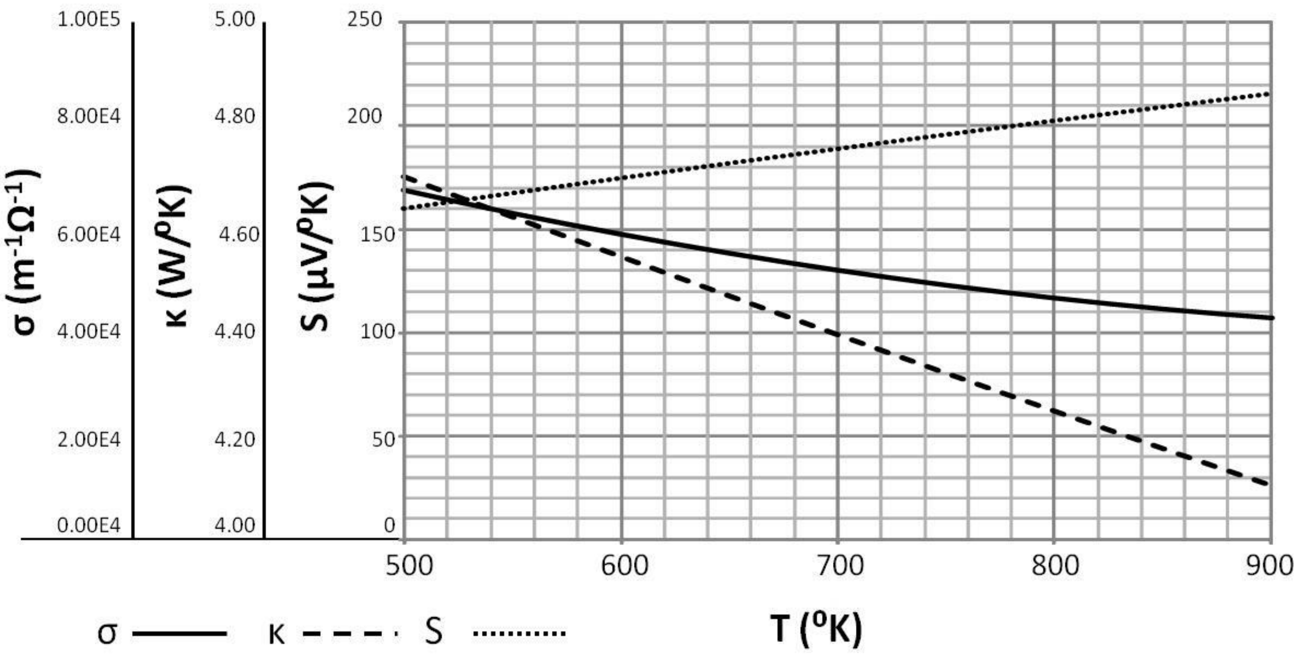

Figure 4 – Thermoelectric Properties for P-Type SiGe [4]

Using a value for the substrate conductivity of κ_S = 2 W/mK and a value for the conductor thickness of L3=0.2 mm, we generate a family of curves for maximum power and efficiency as functions of the thermoelectric thickness L_1 and length L_2. These are shown in Figure 5. From Figure 5(a), for all pellet thicknesses L_1, maximum power is seen to initially increase with pellet length L_2. In Figure 5(a), the maximum power for all combinations shown is 15 mW and occurs when L_1 = 16 mm and L_2 = 16 mm. However, a better strategy for design is guided by the rate of change of maximum power. For all curves shown in Figure 5(a), the initial slope denotes the maximum rate of increase in power with length. The dashed line in Figure 5(a) depicts the steepest such slope and has the equation Power = 2.0413•L_2. When the curve begins to deviate from this slope, rather than use a longer pellet, a better choice is to add an additional pellet. For example, for the case where L_1=16 mm (a rather thick choice), a one-element generator of length L_2=16 mm will produce 15 mW of electrical power. However, a pellet of length 4 mm will yield 5.5 mW, so four of these shorter pellets still adds up to 16 mm of length but can produce 22 mW of power[1]. The highest initial slope in Figure 5(a) corresponds to a pellet thickness of L_1=4 mm, which makes this the best choice for power generation. From Figure 5(b) we note that thicker thermoelectric elements always result in higher conversion efficiencies from heat energy to electrical energy.

1 We ignore the "overhangs" between pellets. These can be significant and represent inert regions with no power generation. As such, the best choice of L_2 may be a choice which is a little longer and departs slighly from the dashed "maximum" power line in order to reduce the proportional influence of the overhangs.

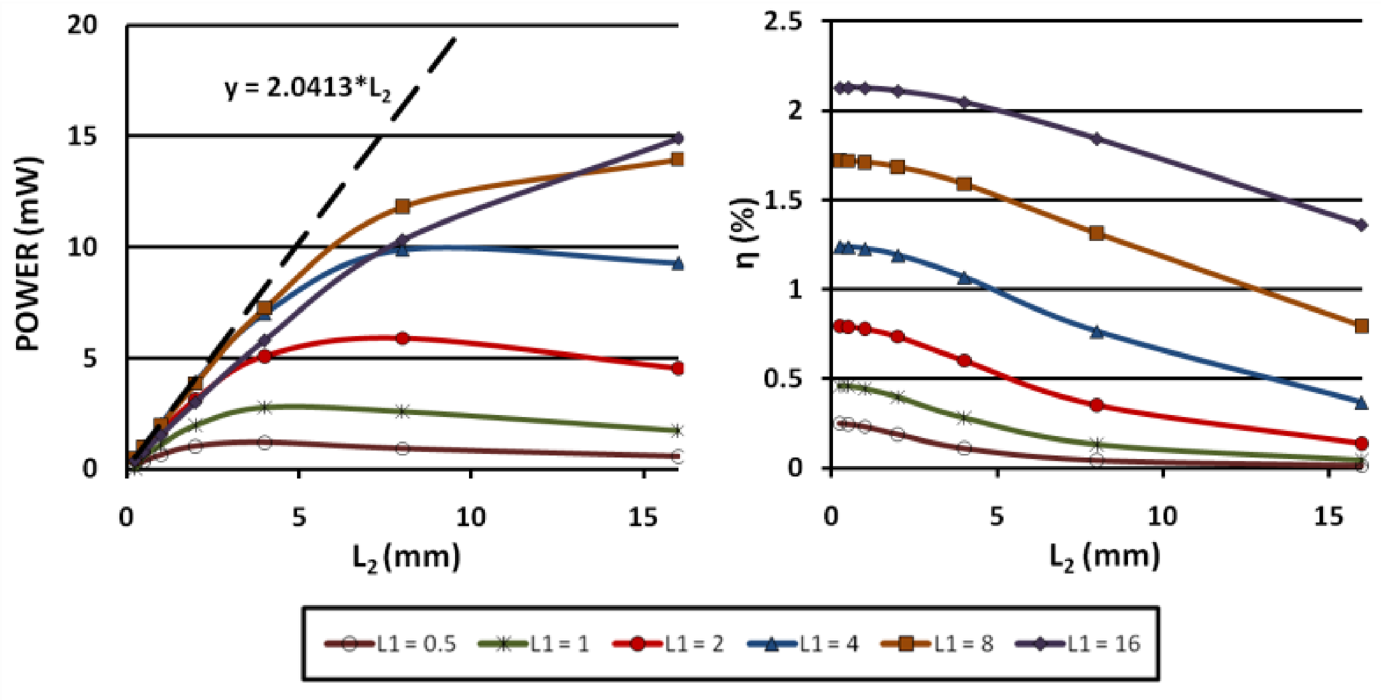

Figure 5 – Maximum Power and Conversion Efficiency as a Function of Thermoelement Thickness and Length Using P-Type SiGe and Conductor Thickness L_3=0.2 mm

The Figure 5 plot expresses the performance when the conductor thickness is L_3=0.2 mm. To understand the role that conductor thickness plays, maximum generated power data was generated for the L_3=0.2, 0.4, 0.8 and 1.6 mm cases and is shown in Figure 6(a-d). In each of these plots, a dashed line corresponding to maximum initial power increase with respect to thermoelectric length, L_2. We note that of plots a-d, the highest slope for the dashed lines occurs for the smallest conductor thickness, (L_3=0.2 mm) and that slope decreases slightly as the conductor thickness increases. This is due to the slight increase in thermal parasitics in the conductors, causing a reduction in the temperature gradient across the thermoelement. An important observation is that the departure of the power curves from the maximum power slope (dashed line) occurs at longer thermoelectric lengths when the conductor thickness is greater. So, having thicker conductors detracts slightly from power, as seen from the lessening of maximum power slopes, but has the advantage that thermoelectric elements may be designed to be longer before a diminishing return on power production is observed.

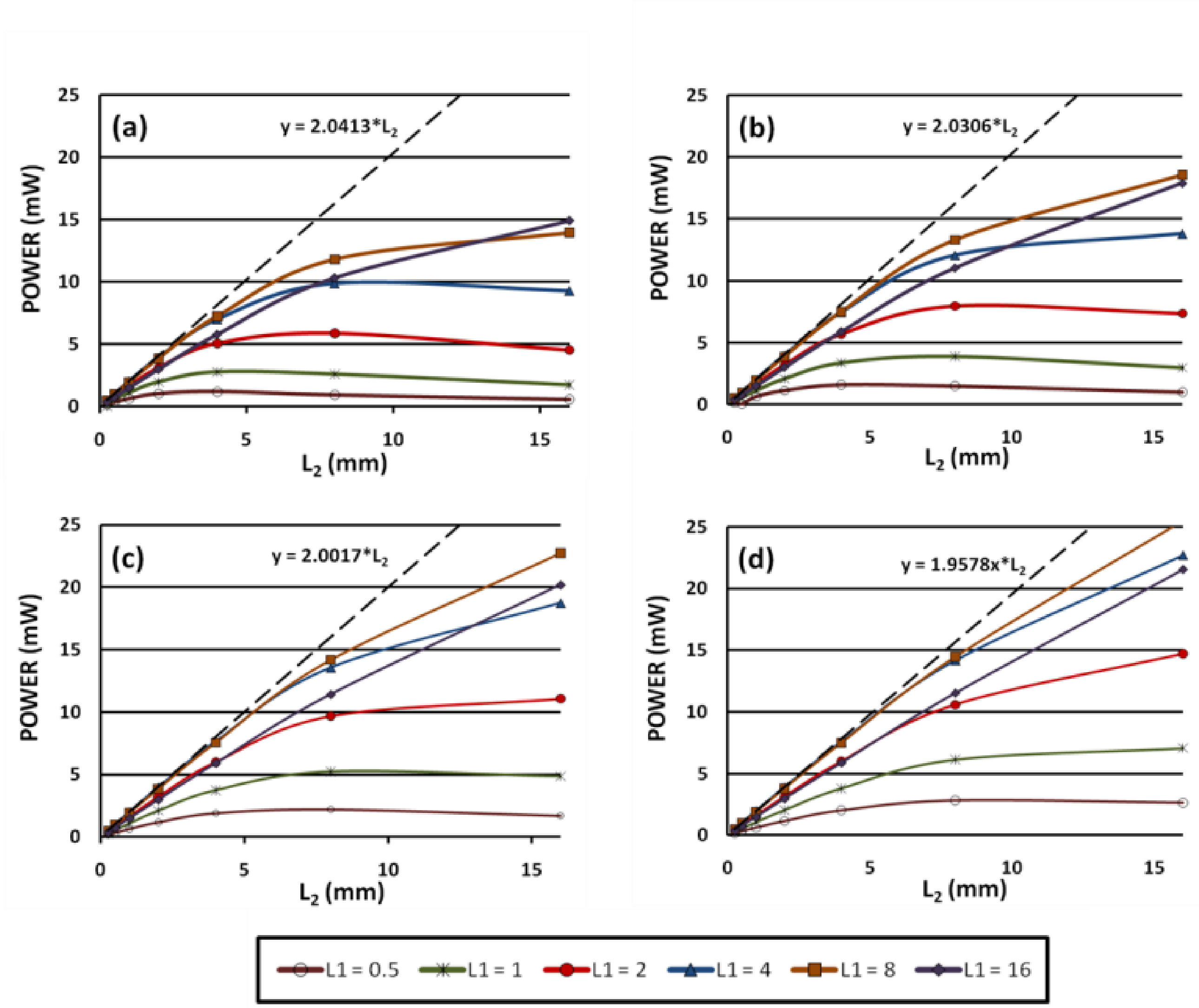

Figure 6 – Power Generation as a Function of Dimension Using a P-Type SiGe Thermoelectric for (a) L_3=0.2 mm; (b) 0.4 mm; (c) 0.8 mm; (d) 1.6 mm

Equation (3), which defines the maximum power that can be produced from a thermoelectric pellet, suggests that power scales with pellet area. For a pellet of fixed depth (1 mm into the page for our present study), this would mean that doubling the length of the pellet will result in double the power. Equation (1) defines the efficiency with which electrical power can be produced and it is independent of the dimensions of the pellet. Clearly, from Figure 5, neither of these idealized results in equations (1) and (3) is achieved in practice and the difference is due to inefficiencies introduced by connection elements. For a given thermoelectric material/electrical conductor choice, the electrical power that is produced, and the efficiency with which that electrical power is produced, are compromised by the connection elements. We call these effects "parasitics" since they reduce performance. Electrical parasitics arise from Joule losses both within the thermoelement and in the electrical conductors. Thermal parasitics are the temperature drops in the substrates and conductors that reduce the ΔT in Figure 3. These parasitics are distributed quantities that arise through the coupled interaction of electrical current and heat flux in the generator. One other indirect source of loss, is the inert "overhang" of conductor that attaches adjacent thermoelements. Table 1 summarizes some guidelines for designing thermoelectric generators to reduce the parasitics.

If	**THERMAL PARASITICS**	**ELECTRICAL PARASITICS**
L1 INCREASES	**DECREASE** More temperature drop across thermoelement	**INCREASE** Longer electrical path from A to B (Fig 3) in each element
L2 INCREASES	**DECREASE** Desensitizes impact of overhang between elements	**INCREASE** Longer electrical path from A to B (FIG 3) in each element
L3 INCREASES	**INCREASE** Additional resistance between reservoirs	**DECREASE** Lower resistance in conductors.

Table 1 – Designing to Reduce Electrical and Thermal Parasitics

CONCLUSIONS

A thermoelectric generator that is based upon idealized interconnections (eg: perfect conductors, zero thermal resistance between generator and thermal reservoirs) has great flexibility in design. However, practical connections result in limitations on the element design. In the present study, using thicker electrical conductors allows the use of longer thermelectric elements but this will not always be true since if these conductors are thick enough to develop significant temperature drops it can cause a reduction in power production. Thicker thermoelectric elements always improves efficiency but there is an optimal thickness from the standpoint of power production and elements that are thicker or thinner than the optimal will yield less power.

When thermoelectric materials are interconnected to form a generator, electrical and thermal parasitics have a major influence on performance and impact both the design and the preferred thermoelectric material. Reducing parasitics in a design is always desirable, but when power production levels are low, electrical parasitics are less significant than thermal parasitics. When a thermal reservoir is limited (eg: high thermal resistance between T_H and T_C and the thermoelements, then a thick thermoelement allows a higher efficiency of electrical generation with minimum impact on overall power production. When the thermal reservoirs are more substantial, thinner thermoelements allow an overall higher power production but with a lower conversion efficiency.

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